$SU(N_c)$ gauge theories at large-N_c from the lattice

Barak Bringoltz

With M. Teper and A. Athenodorou from the University of Oxford:

BB and Teper, in progress and arxiv:0709.2981

Athenodorou, BB and Teper, Physics Letter B Sep '07

BB and Teper, Physics Letter B Nov '06

Why study large-N_c ?

Goal : understand (solve ?) **Q**uantum **C**hromo **D**ynamics nonperturbatively

QCD

- $SU(3)$ gauge symmetry, quarks of 3 colors.
- High energy : asymptotically free.
- Confining spectrum :

  mesons, baryons, glueballs.
- **Nontrivial dynamics :**

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't Hooft '72 : N_c-QCD

- $SU(N_c)$ gauge symmetry, quarks of N_c colors.
- High energy : asymptotically free. $\surd$
- (Believed to have) Confining spectrum :
  mesons, baryons, glueballs. $\surd$
- **Much simpler dynamics :**
  scatterings, mixings, decays $\mathcal{O}\left(1/N_c\right) \xrightarrow{N_c \to \infty} 0$

Large-N_c spectrum $+$ perturbations in $\frac{1}{N_c}$ $=$ predictions for QCD spectrum

Why study large-N_c on the lattice ?

Despite this, QCD at large-N_c has not been solved. (yet).

$\longrightarrow$ **This makes a strong case for lattice explorations of large-N_c QCD.**

1. **Phenomenology :** provide $N_c = \infty$ starting point for $\frac{1}{N_c}$ expansion. Steve's talk
2. **Practically :** simpler theory $\rightarrow$ cleaner analysis :
   - Theoretical uncertainties in interpreting lattice data go away at large-N_c.

     $\Downarrow$

   - Cleaner and more confident predictions at large-N_c $\longrightarrow$ infer on QCD.
3. **Theoretically (A) :** test and guide analytic progress
   - AdS/CFT.
   - Karabali-Nair approach.
4. **Theoretically (B) :** $N_c > 3$ opens up “hidden” and revealing physics.

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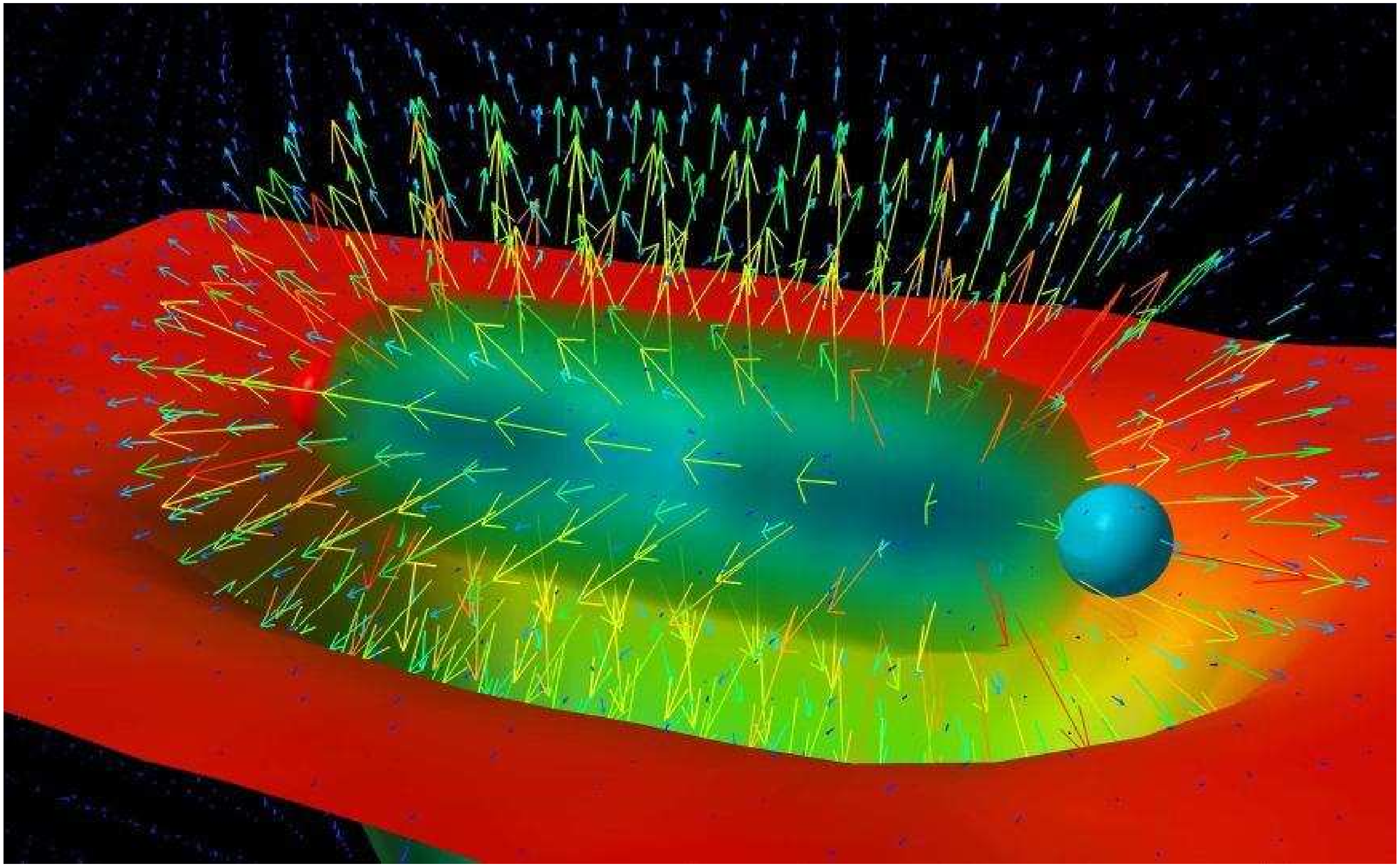
A single example : Confining flux-tubes in $2+1$ dimensions.

BB and Teper – Phys.Lett.B, Nov '06,

Athenodorou, BB and Teper – Phys. Lett. B, Sep. '07.

Confining flux-tubes

Presence of confining flux-tubes in the vacuum = defining property of QCD :



$SU(3)$, pure gauge, $l \simeq 1.5$ fm, Leinweber et al.

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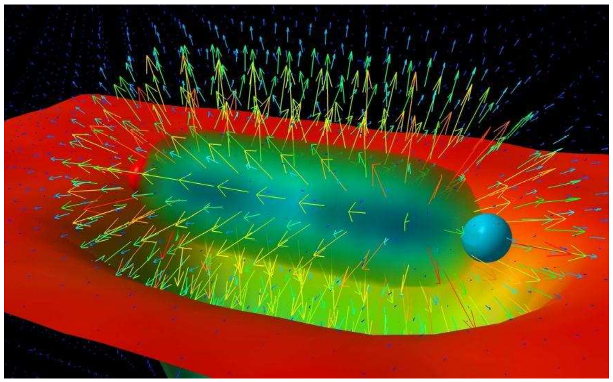
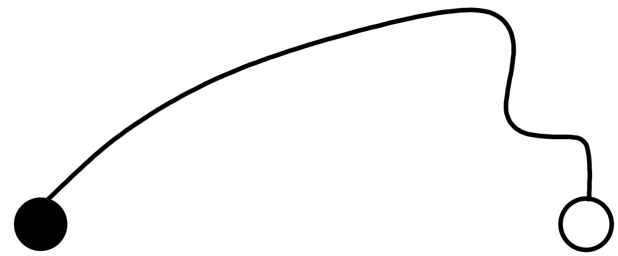


$l \simeq 1.5$ fm, Leinweber et al.

$l \gg w$

$\simeq$



Mathematically :

$$S = S_{\text{effective}-\text{string}} \quad \text{for} \quad l \gg l_{min} \simeq w$$

Want to know properties of $S_{\text{effective}-\text{string}}$:

- Low energy degrees of freedom ?
- Form of interactions ?
- What is $\boldsymbol{l_{min}}$? $\boldsymbol{2w}$? $\boldsymbol{100w}$?

Confining flux-tubes

Properties of $S_{\text{effective}-\text{string}}$ reflected in

$$E_n(l) = \sigma\, l + \text{sub leading}$$

- "sub leading" are very small ! **5% at $\simeq 1.5$ fm**
- $E_n \simeq \sigma\, l$ are large, signal exponentially small $\sim e^{-E_n\, t} \simeq e^{-\sigma\, l\, t}$.

In practice use moderately-large/short l, which is has theoretical uncertainties :

Is $l > l_{min}$? Are you seeing the a string, or some short-l physics ?

⇓ Glueball-string mixing ?

Study large-N_c and compare to QCD :

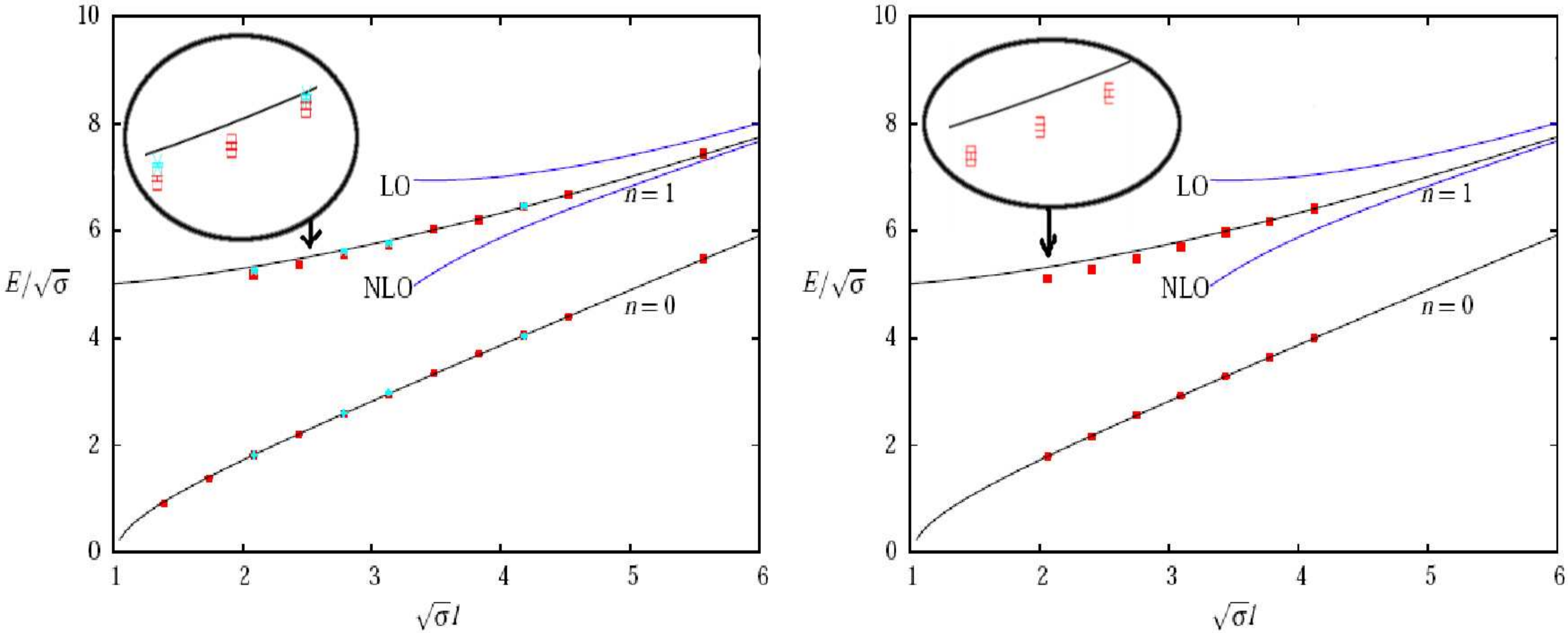
✗ Different $\longrightarrow$ measurement contaminated by short-l physics.

√ Similar $\longrightarrow$ you have a string.

Confining flux-tubes

Spectrum in $SU(2,3,4,5,6,8)$ gauge theory, ~ 30 lowest states :

$SU(3)$ $SU(6)$



$\rightarrow$ No ambiguities w/ large-N_c $\rightarrow$ fit $E_0(l) = \sigma l + \text{sub leading}$ reliably

$\rightarrow$ Large-N_c flux-tube $\simeq$ relativistic string : guides analytic large-N_c Aharony et al. (AdS/CFT)

Confining flux-tubes

Karabali-Nair and Co. ‘95-‘98, ‘06-‘07 :

$$\boxed{\frac{\sigma_{2+1}}{(g^2 N_c)^2} = \frac{1}{8\pi} \times \left(1 - 1/N_c^2\right)}$$ Karabali-Kim-Nair ‘98

Remarkably :

$$\sigma_{\mathrm{KKN98}} - \sigma_{\mathrm{lattice}} = (3\,\% - 1.14\,\%) \xrightarrow{N_c \to \infty} 0.96\,\%$$ Teper and Lucini ‘01

$\longrightarrow$ Is this real or a systematic error? Did Karabali-Nair solved confinement ?

If so, this is significant progress in field theory

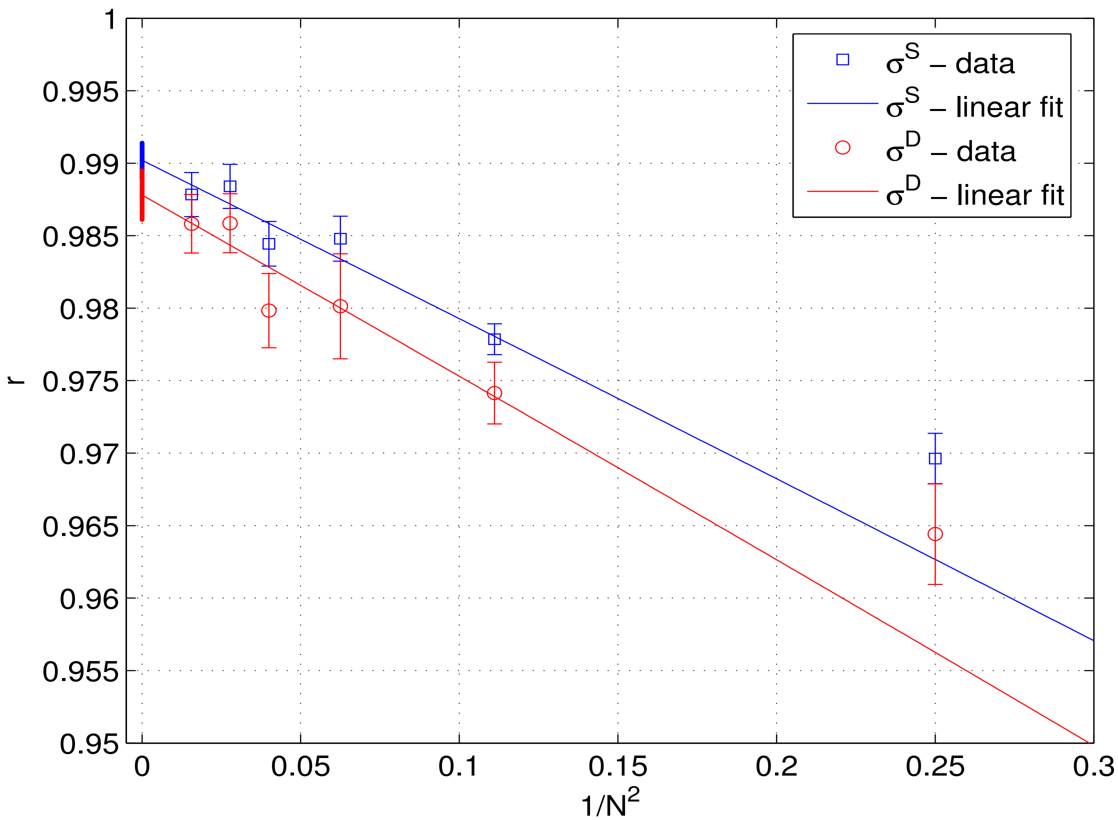
Lattice needs a fraction of a percent accuracy (!) **Which we managed to do**

knowing the “sub leading” in $E(l)$ was only one step to achieve this ...

Confining flux-tubes

After continuum & infinite volume extrapolations.

Using our accurate fit of $E_0(l) = \sigma l + \text{sub leading}$.



- $r \equiv$ Lattice/KKN98.
- $\frac{\sqrt{\sigma}}{g^2 N} = \mathbf{0.1975 \pm 0.0002 - 0.0005}$

  vs.

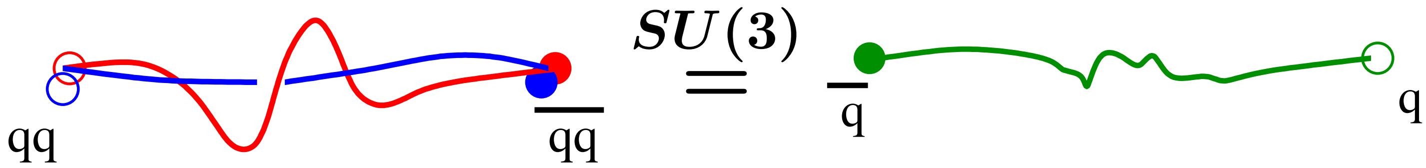
  $\frac{1}{\sqrt{8\pi}} \simeq \mathbf{0.1995\ldots}$

1.013 % = 6-10 sigma

→ **Large-N_c calculations are essential to test analytic methods.**

Current project : bound states of strings

Consider the flux between qq and $\bar{q}\bar{q}$:



For $N_c > 3$ no longer true :

- Is there a loosely/tightly bound state ?
- What is it ? a new string ? What is its S_{eff} ?

Despite absence in QCD : this study tests & expands our knowledge of confinement:

E.g., preliminary results:

- Surprising N_c dependence of binding energy.
- Surprising excited state spectrum.

$\longrightarrow$ **Large-N_c uncovers 'hidden' QCD sectors which are revealing.**

Future projects

Bound states of strings in $2+1$ dimensions : with Teper & Athenodorou

- How far is KKN98 ?
- Use our methods to extract more complete spectrum (tens of lowest states).

Bound states of strings in $2+1$ dimensions : with Yaffe

- Build a model to explain spectrum.

Precision measurements of Glueball masses : with Teper

- To guide Karabali-Nair & AdS/CFT approaches.

String spectrum in $3+1$: with Teper & Athenodorou

- Relativistic string as well ?

Large-N_c reduction, large-N_c meson spectrum : with Sharpe and Yaffe

Large-N_c QCD thermodynamics: actively studied via AdS/CFT (Karch, Son, Yaffe)

Conclusions

Large-N_c explorations of lattice QCD are useful for many reasons :

- **Can help us understand experimental spectrum.**
- **Removes theoretical uncertainties from lattice data.**
- **Tests & guides analytic approaches.**
- **Opens windows to hidden QCD sectors.**

$$\Downarrow$$

Instrumental for our understanding of QCD.