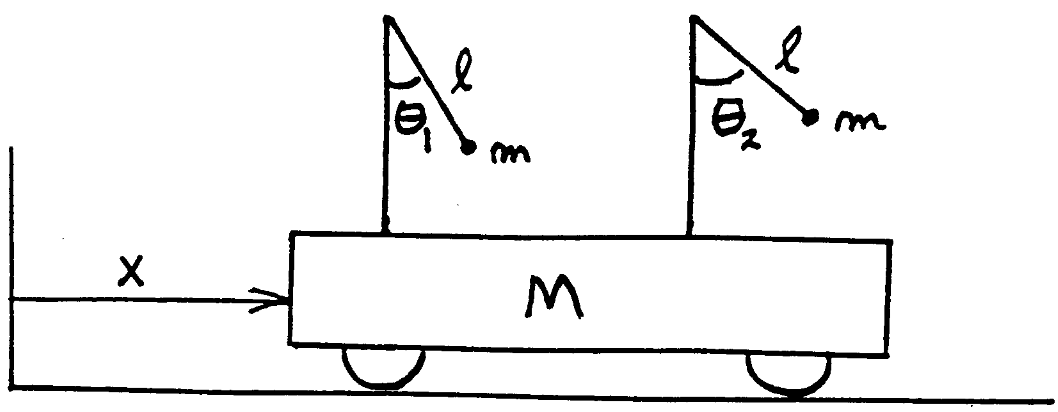
CLASSICAL MECHANICS

Autumn-98 Qualifying Examination

1. Inertially Coupled Pendula

We shall work out the motion of two pendula which are attached to a "railroad car" that can move without friction along a straight line. We approximate the railroad car by just a simple mass M and neglect the moments of inertia of the wheels. The two pendula are constrained to swing only in the plane that contains the straight line along which the car moves. The two pendula are identical: They both have a mass m attached to a rigid rod of length l whose mass can be neglected, with the rod pivoted on a frictionless pivot. The force of gravity mg acts downward on both of the pendula masses. We denote the position of the car along the line by x and the angles that the two pendula rods make with the vertical by θ_1 and θ_2. Here is a sketch of this mechanical system:



a) **15 pts.** Write down the Lagrangian of this system.

From now on. make the small angle approximation $\theta_1 \ll 1$, $\theta_2 \ll 1$.

b) **15 pts.** Derive the Lagrange equations of motion.

c) **20 pts.** Solve the Lagrange equations of motion for the two normal modes of the system.

2. Safe Driving

In this problem we shall consider some motions of an automobile which has a wheelbase (the distance between front and rear axles) l and whose center-of-mass is half way between the front and rear axles and a distance h above the ground.

a) **20 pts.** Suppose that the driver is braking the car. Sketch and label all the forces acting on the car and where they act. Suppose now that the braking results in a de-acceleration b of the car. Let x be the fraction of the weight of the car carried by the front wheels so $1 - x$ is the fraction of the weight carried by the back wheels. Prove that

$$x = \frac{1}{2} + \frac{b}{g}\frac{h}{l},$$

where g is the acceleration of gravity at the surface of the earth.

b) **15 pts.** Suppose now that the car is going about an (un-banked) curve of radius R (assume $R \gg l$) at speed v. There is a maximum speed v_{max} that the car can go about this circle before the tires slip. Show that v_{max} is given by

$$v_{\mathrm{max}}^2 = \mu g R\,,$$

where μ is the coefficient of static friction of the tires. In view of the fact that the car has a finite moment of inertia about the vertical axis through the center of mass. show that this result does not require that the center of mass be exactly half way between the front and rear car axles. Be sure to EXPLAIN YOUR REASONING.

c) **15 pts.** But now suppose that the car is going about the curve at a speed that is just a little less than v_{max} and the driver becomes nervous and applies the brakes fairly hard. What happens? Do not give precise quantitative results. but be sure to EXPLAIN YOUR REASONING.