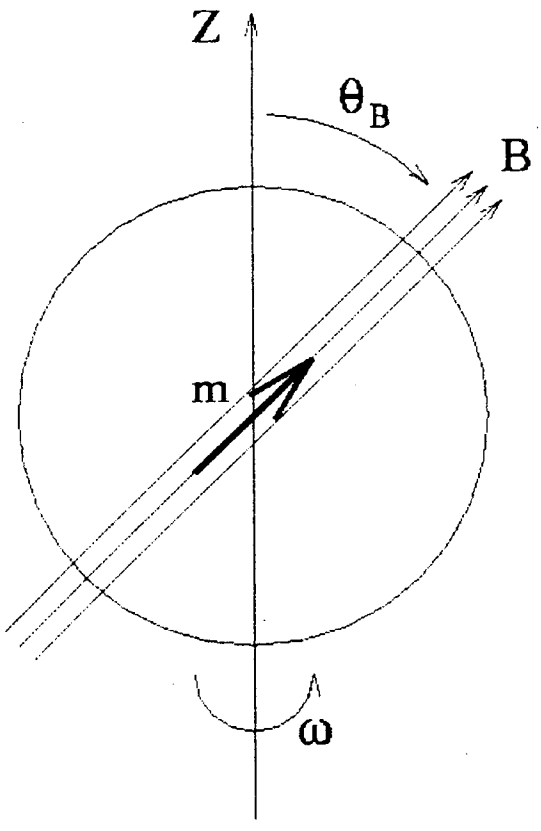
Electricity and Magnetism - Aut '97 Qual



Rapidly rotating neutron stars often have strong magnetic moments. A considerable energy flux can be carried off by electromagnetic radiation, powered by the rotational energy of the neutron star.

Suppose that the magnetic moment axis is *not* aligned with the rotation axis, so that the neutron star has a magnetic dipole moment $\vec{m}$, which changes in direction with time. Let θ_B denote the angle between the magnetic and rotation axes and let ω denote the rotation angular velocity.

(a) [30 pts.] Derive an expression for the angular dependence of the emitted radiation in the "far" wave zone, for the radiation due to the changing magnetic dipole. Sketch the angular dependence in the wave zone of the emitted power in a plane that includes the star's rotation axis.

(b) [20 pts.] Describe the direction of the $\vec{B}$ field of the radiation in the "far" wave zone observed in a direction $\vec{n}$ from the source. What is the direction of the corresponding electric vector $\vec{E}$? Describe the polarization of radiation propagating along the rotational axis.

(c) [10 pts.] What is the frequency spectrum of the emitted radiation? For rotational frequencies $\omega \approx 10^3$ Hz, what happens to this radiation if the source is embedded in ionized gas which has a plasma frequency of 10^6 Hz? With a plasma frequency of 1 Hz?

(d) [30 pts.] Suppose that the surface magnetic field strength at the magnetic pole is $B_0 = \frac{2(|\vec{m}|)}{R^3}$ where R is the neutron star's radius. Find an expression for total radiated power P in terms of c, ω, R, B_0, and θ_B.

(e) [10 pts.] Assume that this power is extracted from rotational energy which for a homogeneous sphere of mass M is $E_{rot} = (2/5)MR^2\omega^2$. Find an expression for the "spin-down" time scale $\tau \equiv -\frac{\omega}{\dot{\omega}}$, in terms of M, R, ω, B_0, θ_B, and c.