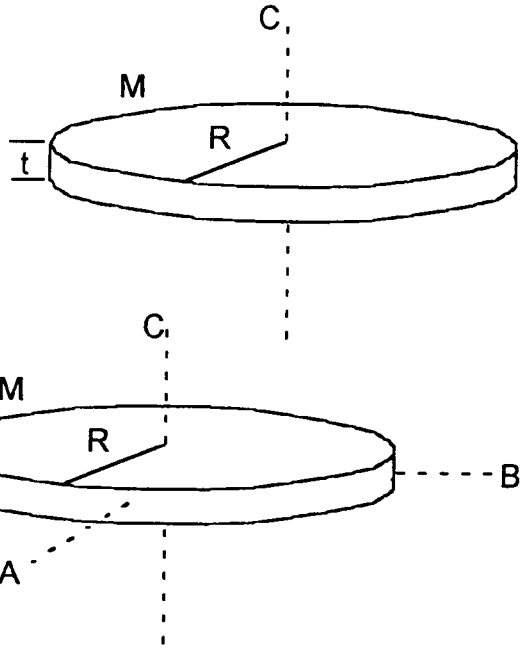
Classical Mechanics - Qual - Aut '97

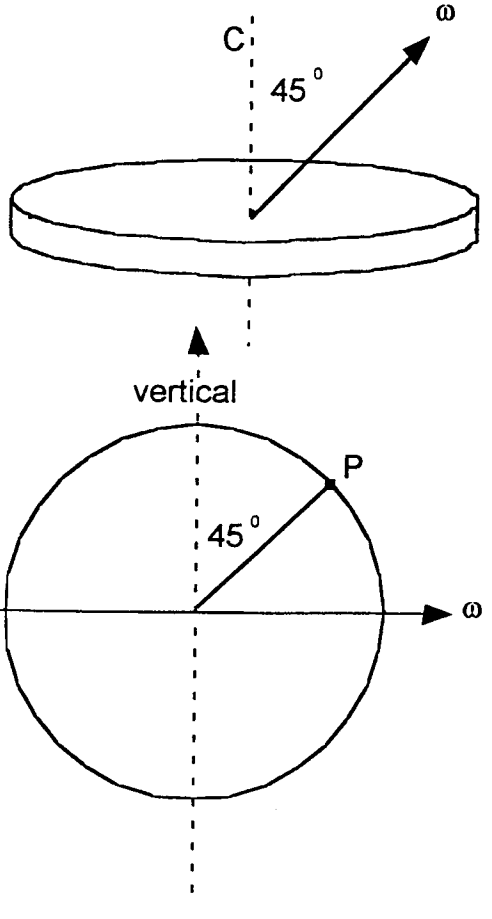


A . Consider a uniform density disk of radius R, thickness t, and mass M.

i. [10 pts] Calculate the moment of inertia, I_C, about the symmetry axis, C (dotted line in diagram).

ii. [15 pts] For a lamina, the perpendicular axis theorem states that the sum of the two moments of inertia about perpendicular axes in the lamina is equal to the moment of inertia about an axis through the intersection of the two axes and perpendicular to the lamina. Prove this theorem and use it to calculate the moments of inertia, I_A and I_B about axes A and B in the limit $t << R$.

iii. [10 pts] Axes A, B, and C are the principal axes of the moment of inertia tensor referred to the center of mass. Explain why this is so.



B. The disk is set spinning with angular velocity ω about an axis through the center of mass that makes an angle of 45° with respect to the symmetry axis C. The axis is vertical.

i. [15 pts] Calculate the angular momentum L of the disk

ii. [20 pts] Describe the subsequent motion. Neglect gravity.

C. The disk is now flipped vertically into the air spinning with a horizontal angular velocity ω about axis B. At the top of its trajectory it strikes an object that instantaneously brings to rest a point P on its rim. At this moment the plane surface of the disk is vertical (as shown) with a radius to the point P making an angle of 45° with respect to the vertical.

i. [10 pts] Show that the change in angular momentum is equal to the impulsive torque and calculate the impulsive torque about P.

ii. [20 pts] Use part C*i.*, calculate the angular velocity ω′ and the velocity of the center of mass v_{CM} immediately after impact.