

Spv 96

## Basic Physics

### 1. (76 points) *Gravitational Orbits*

Kepler's 1st Law states that all planets move in elliptical orbits having the Sun at one focus. Kepler's 2nd Law states that a line joining any planet to the Sun sweeps out equal areas in equal times. Kepler's 3rd Law states that the square of the period of any planet about the Sun is proportional to the cube of the planet's mean distance from the Sun. Consider a planet of mass  $M_p$  in an orbit around the Sun, which has a mass  $M_s$ . Assume that  $M_s \gg M_p$ .

(a) First, assume that the planet is in a circular orbit of radius  $R$ . Calculate the planet's orbital period  $T$  and relate your result to Kepler's 3rd law. Explain qualitatively why  $T$  does not depend on  $M_p$ .

(b) For the above case, calculate the ratio of the kinetic energy of the planet in orbit to its potential energy. Would you expect the same ratio for elliptical orbits?

(c) Now assume that the planet is in any elliptical orbit and find its angular momentum  $L$  in terms of its instantaneous angular velocity  $\omega$ . Use this to derive Kepler's 2nd law.

(d) A weakly interacting massive particle (WIMP) of mass  $M_w$ , which interacts with normal matter only gravitationally, is released at rest at the Earth's surface and moves through the Earth's gravitational field. Assume that the Earth is a sphere of uniform density, mass  $M_e$ , and radius  $R_e$ , and ignore the Earth's rotation. Show that the gravitational force acting on the WIMP is a Hook's Law force (i.e., proportional to  $-r$ , where  $r$  is the distance from the center of the Earth), and calculate transit time  $T_w$  of the WIMP through the Earth from one side to the other. How would you expect  $T_w$  to be modified if the calculation were done with a more realistic non-uniform density?

### 2. (24 points) *General Qualitative Questions*

*The following questions are intended to test your qualitative understanding. Answer each with a few short sentences.*

(a) For a planet in an elliptical orbit, briefly describe the direction and changes in direction of its velocity, acceleration, and angular momentum.

(b) NASA wishes to train astronauts by subjecting them to a few minutes of weightlessness. Describe how this might be (and is) accomplished by flying an airplane in a particular way.

(c) Briefly explain what the Michaelson-Morely experiment was attempting to measure and how. Why did it fail?