Aug
~~Spring~~ 2006 Qualifying Examination – Electromagnetism

1. [30 points] Consider an insulated spherical conductor of radius a, and a point charge q at a distance $r>a$ from the center.
   a. Show that the effect of the conductor is equivalent to point charges qa/r at the center of the conductor, and $-qa/r$ at a distance a^2/r along the line from the center to the point charge.
   b. What is the smallest positive charge that can be applied to the sphere such that the surface charge density on the sphere is everywhere positive?

2. [40 points] A beam of plane polarized electromagnetic radiation of frequency ω, electric field amplitude E_0 is normally incident on a region of space containing a low density, neutral plasma, n_0 electrons/unit volume.
   a. Calculate the conductivity as a function of frequency.
   b. Show that the dispersion relation has the form, $\omega^2 = k^2c^2 + \omega_p^2$, identifying the plasma frequency ω_p, and determine the index of refraction.
   c. A pulsar emits a pulse of broadband electromagnetic radiation which is 1 ms in duration. The pulse then propagates 1000 light years (10^{21}) cm through interstellar space to reach radio astronomers on Earth. Assume that the interstellar medium contains a low density plasma with plasma frequency ω_p = 5000 s^{-1}. Estimate the difference in measured pulse arrival times for radio telescopes operating at 400 MHz and 1000 MHz.

3. [30 points] A Hall probe with dimensions as shown has conductivity σ and carries charge density ρ. The probe is placed in an unknown magnetic field B oriented in the +y direction. An external potential V_{ext} is applied to two ends producing an electric field in the +z direction. Between which pair of ends is the equilibrium Hall voltage V_{Hall} observed? Derive an expression for B involving the above quantities and the dimensions.