Fall 2006 Qualifying Examination – Classical Mechanics

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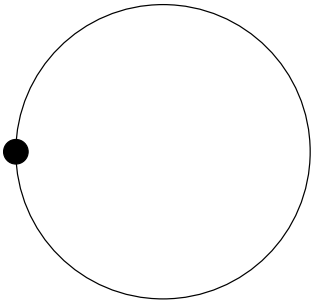
1. **[60 points total]. Motion in central potential**

A particle moves in a central potential of the form $V(r) = -C/r^\alpha$, where C and α are constant.

(i) [**20 points**] For which values of α are there stable circular orbits in the potential $V(r) = -C/r^\alpha$?

(ii) [**20 points**] It is known that during a portion of its trajectory, the particle moves along a circle of radius R that goes *through* the center of gravity.



Let r and θ be the coordinates of the particle in the polar system of coordinates ($r = 0$ being the center of gravity). Using Kepler's second law, give the expressions for the time derivatives of the radius and polar angle, $\dot{r}$ and $\dot{\theta}$, as a function of the polar angle θ and constants of motion.

(iii) [**20 points**] Find α (hint: use energy conservation). Are there stable circular orbits in this potential?

2. **[40 points total]. Small oscillations**

In the simplest mechanical model, the carbon dioxide molecule is a chain O—C—O, and the carbon monoxide molecule is a chain C—O. The mass of the carbon atom is $m_1 = 12$ atomic units, and that of the oxygen atom is $m_2 = 16$ atomic units. The chemical bonds between the carbon and the oxygen atoms are modeled as springs with (identical) spring constant k.

(i) [**5 points**] Restrict yourself to longitudinal motion (in which each atom moves only along the direction of the line connecting the atoms in the molecules). How many modes of oscillation exist for each molecule?

(ii) [**20 points**] Find the frequencies of the longitudinal oscillations of the two molecules.

(iii) [**15 points**] Graphically sketch the longitudinal oscillation modes of the CO_2 molecule.