

Autumn 2006 Qualifying Examination -- Super Basic

Useful Information: $hc = 1.239 \times 10^{-6} \text{ eV m}$, $c = 3.0 \times 10^8 \text{ m/s}$,
 $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2$, $e = 1.60 \times 10^{-19} \text{ C}$, $1/4\pi\epsilon_0 = 9.0 \times 10^9 \text{ N m}^2/\text{C}^2$

1. [40 points total] (Special Relativity – Compton Scattering)

A photon of wavelength λ_0 collides with an electron of mass m_e at rest and scatters by angle θ_1 with a new wavelength λ_1 . The electron recoils and moves away at angle θ_2 with respect to the direction of the initial photon.

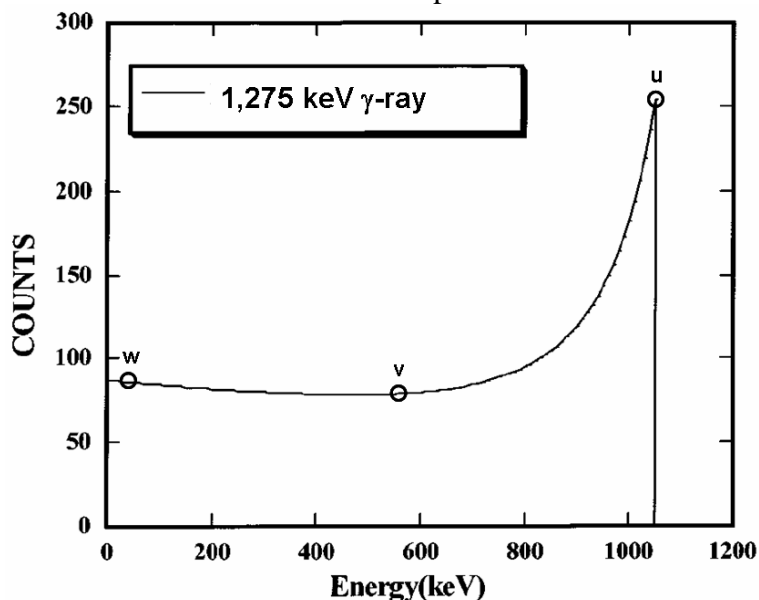
- A. [20 points] Use relativistic kinematics to show that the change in photon wavelength $\Delta\lambda = \lambda_1 - \lambda_0$ as a function of θ_1 is:

$$\Delta\lambda = \frac{h[1 - \cos(\theta_1)]}{m_e c}.$$

- B. [10 points] Show that the kinetic energy K of recoil of the electron is:

$$K = \frac{hc[1 - \cos(\theta_1)]}{\lambda_0 \left[1 + \frac{m_e c \lambda_0}{h} - \cos(\theta_1) \right]}$$

- C. [10 points] A plastic scintillator is irradiated with 1.275 MeV gamma rays, and the calibrated intensities of the scintillation flashes are measured. The resulting scintillation spectrum is shown below. Relate the equation derived in part B above to points u, v, and w of the observed scintillation spectrum.



2. [30 points total] (Newtonian Mechanics)

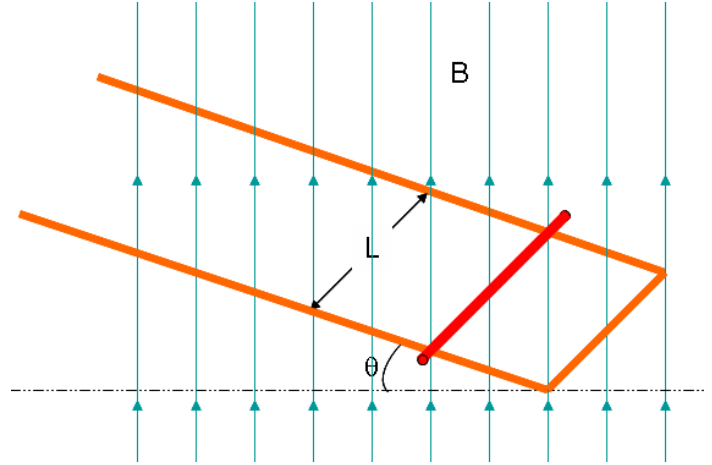
A particle of mass m is subject to the constant pull of gravity with acceleration g and to the force of air resistance. The particle is dropped from point $x = 0$ and $z = z_0$. The initial velocity is zero. The force of air resistance is assumed to be linearly dependent on the speed. The motion of the projectile $z(t)$ is described by the equation:

$$mz'' = -mg - bz'$$

- A. (20 points) Solve the equation of motion for the particle's position $z(t)$ if the initial conditions are $z'(0) = 0$ and $z(0) = z_0$. Find the terminal velocity.
- B. (5 points) Take the limit as $b \rightarrow 0$ (i.e., no air resistance) and show that the free-fall solution is obtained.
- C. (5 points) Make a rough plot of the velocity of the particle with $b=0$ and $b=0.2$ kg/m in the time interval from $t=0$ to $t=10$ s for the values $m= 1$ kg and $g=10$ m/s.

3. [30 points] (E&M)

A rod of mass m and resistance R slides without friction down parallel conducting rails of negligible resistance spaced a distance L apart and making an angle θ with the horizontal, as shown. The ends of the rails are connected at the bottom end. The system is in a uniform gravitational field g and a vertical uniform magnetic field B .



A. [5 points] Find the total electrical force on the rod, if it is moving down the rails at velocity v .

B. [10 points] Show that the rod acquires a steady-state terminal velocity

given by:
$$v = \frac{mgR \sin \theta}{B^2 L^2 \cos^2 \theta}$$

C. [10 points] Show that the rate at which internal energy is being generated by the rod is equal to the rate at which the rod is losing gravitational energy.

D. [5 points] Discuss the behavior of the rod and the energy flow if the magnetic field were in the opposite direction.