Autumn 2005 Qualifying Examination -- Super Basic

Useful Information: $\hbar = 1.0546 \times 10^{-34}$ J s, $c = 2.998 \times 10^{8}$ m/s, $\mu_0 = 4\pi \times 10^{-7}$ N/A^2, $e = 1.602 \times 10^{-19}$ C, $1/4\pi\varepsilon_0 = 8.99 \times 10^{9}$ N m^2/C^2, $\varepsilon_0 = 8.854 \times 10^{-12}$ C$^{2/}$N m^2, $m_e = 9.11 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg,

1. [30 points total] (**The Bohr Atom)**

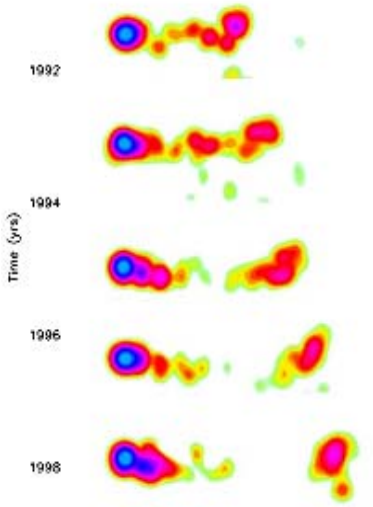
A. (8 points) Show that the circumference of a Bohr orbit is an integer multiple of the de Broglie wavelength of the orbiting electron.

B. (12 points) Using dye lasers, hydrogen atoms with electron orbits having principal quantum number of **n**=600 have been produced in the laboratory. What would be the diameter of such a "Rydberg" atom and what would be the transition energy to an adjacent Bohr orbit?

C. (10 points) Calculate the classical velocity of an electron in the **k**-shell (**n** = 1) of a hydrogen-like atom with a nuclear charge of **Z**=144, and comment on the significance of its magnitude.

2. [30 points total] (**Special Relativity – Superluminal Objects)**

Radio telescope observations of quasars show many with blobs of radio-emitting gas that appear to be separating from the quasar at up to 8 light years per year of observation (i.e., 8 times the speed of light). This has been shown to be a geometrical illusion arising because the blob is moving almost directly toward us along the line of sight.



A [10 points] Show that the apparent velocity of the object is $\beta_{app} = \beta \sin\theta/(1 - \beta \cos\theta)$ where θ is the angle that the object's velocity makes with the line of sight in the direction of the Earth, and examine the implications of this relation for large β and small θ.

B [10 points] If the blob of gas in the picture is emitted at an angle θ =2 degrees in the Earth reference frame and has an apparent separation velocity of β_{app} = 8c, how much time passes in the rest frame of the blob for the events in the picture?

C [10 points] If radio waves are observed with a frequency of 100 MHz, with what frequency were they emitted, in the rest frame of the blob?

3. [40 points total] (**Magnetic Monopoles)**

A meteorite strikes the Earth and is found to contain a rich supply of magnetic monopoles. These are delivered to your laboratory, where a loop magnetometer is used to determine that they have the magnetic charge **g** predicted for Dirac monopoles. (In 1931, Dirac showed that the minimum magnetic charge that a magnetic monopole could have is $\mathbf{g = 4\pi\hbar/2e = eZ_0/2\alpha}$, where **e** is the electron charge, $\mathbf{Z_0} =$ [illegible] $\mathbf{= 376.7\ \Omega}$ is the impedance of free space, and $\boldsymbol{\alpha = e^2/4\pi\varepsilon_0\hbar c \approx 1/137}$ is the fine structure constant.)

A. (6 points) In the SI system of units, what is the value (with units) of the magnetic monopole charge **g**?

B. (10 points) Find the equivalent of Coulomb's Law in SI units for magnetic monopoles and calculate the ratio of the magnetic force between two monopoles to the electric force between two protons, when both pairs of charged particles are separated by the same distance. Comment on the significance of the size of this ratio for monopole pair creation and cosmology.

C. (14 points) Write Maxwell's equations and the Lorentz force for electric and magnetic charges in SI units and indicate at least three places where they would have to be modified to accommodate this discovery.

D. (10 points) Calculate the ***magnetic*** fine structure constant α_G and comment on the significance of its magnitude.

E. (*extra credit*) Suggest a workable method for measuring the ***rest mass*** of one of these magnetic monopoles.