Spring 2004 Qualifying Examination - Electromagnetism

IMPORTANT: Answer BOTH Questions

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1. [**40** points total] (**Maxwell's Equations and Electromagnetic Fields**)

A. [**5** points] Write the differential form of Maxwell's equations describing electric and magnetic fields in the presence of arbitrary charge distributions and arbitrary current distributions.

B. [**5** points] How are the electric and magnetic fields related to the scalar potential $\phi(\mathbf{r}, t)$ and the vector potential $\mathbf{A}(\mathbf{r}, t)$?

C. [**5** points] If a given system is time-reversed, i.e. $t \rightarrow -t$, what happens to the charge-density, the current-density, the electric field and the magnetic field.

D. [**5** points] If a given system is spatially-inverted, i.e. $\mathbf{x} \rightarrow -\mathbf{x}$, what happens to the charge-density, the current-density, the electric field and the magnetic field.

Consider a system with a vector potential and scalar potential of the form

$$\mathbf{A}(x, y, z, t) \ = \ \frac{B_0}{a}\, xy\ \hat{\mathbf{e}}_z \quad , \quad \phi(x, y, z, t) \ = \ 0 \quad ,$$

where B_0 and a are time-independent constants, and $\hat{\mathbf{e}}_z$ is the unit-vector in the z-direction.

E. [**6** points] Compute the electric and magnetic fields of this system, and sketch representative field lines in the $z = 0$ plane.

F. [**6** points] Do the electric and magnetic fields satisfy the free-space Maxwell's equations?

G. [**8** points] An observer moves with velocity $\mathbf{v} = v\ \mathbf{e}_z$ in the z-direction along the line located at (x, y) in the xy-plane. What is the difference between the scalar potential measured at the point (x, y) and the point $(0, 0)$ in the same z-plane, as measured by the moving observer?

2. [**60** points total] (**Radiation**)

The Lienard-Wiechert form of the electric field resulting from a point particle in motion is given by

$$\mathbf{E}(\mathbf{r}, t) \;=\; \frac{q}{4\pi\epsilon_0}\left[\frac{\hat{\mathbf{R}}_0 - \boldsymbol{\beta}}{\gamma^2 R_0^2(1-\hat{\mathbf{R}}_0\cdot\boldsymbol{\beta})^3} + \frac{\hat{\mathbf{R}}_0 \times \left[(\hat{\mathbf{R}}_0 - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right]}{cR_0(1-\hat{\mathbf{R}}_0\cdot\boldsymbol{\beta})^3}\right]_{\text{retarded}} .$$

A. [**10** points] Briefly explain the meaning of each symbol in this expression for $\mathbf{E}(\mathbf{r}, t)$, and state the implication of the word *retarded* in this context.

B. [**15** points] Show that the power radiated into a solid angle $d\Omega$ is

$$\frac{dP}{d\Omega} \;=\; \frac{q^2}{16\pi^2\epsilon_0 c}\frac{|\hat{\mathbf{n}} \times \left[(\hat{\mathbf{n}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right]|^2}{(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^5} ,$$

and define $\hat{\mathbf{n}}$.

C. [**10** points] For a charged particle accelerating in the direction of its motion, derive $\frac{dP}{d\Omega}$ at an angle θ in the extreme relativistic limit where $\gamma \gg 1$. Sketch the intensity of the radiation in the plane of $\boldsymbol{\beta}$ as a function of θ on a polar plot.

D. [**10** points] For a charged non-relativistic particle moving in a circle of radius R with angular frequency ω, show that the time-averaged power radiated into $d\Omega$ is

$$\frac{dP}{d\Omega} \;=\; \frac{q^2\omega^4 R^2}{16\pi^2\epsilon_0 c^3}\left(1 - \frac{1}{2}\sin^2\theta\right) ,$$

where θ is measured from the symmetry axis perpendicular to the plane of the circle.

E. [**15** points] In Bohr's model of the Hydrogen atom, the electron moves in circular orbits about the proton, and does not radiate unless making transitions between allowed energy-levels. When this system is treated classically it will radiate. Compute the time for the electron to fall from the lowest orbit of radius a_0 into the proton. Use

$$\begin{aligned}
\epsilon_0 &= 8.854 \times 10^{-12} C^2/Nm^2 \\
a_0 &= 5.29 \times 10^{-11} m \\
m_e &= 9.11 \times 10^{-31} kg \\
c &= 2.998 \times 10^{8} m/s \\
e &= 1.60 \times 10^{-19} C
\end{aligned}$$