## Autumn 2002 Qualifying Examination -- Modern Physics

***IMPORTANT:*** Please answer only TWO of the THREE main questions below. Only TWO main questions will be graded. You must cross out the question that you do not wish to be graded.

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1. [50 points total] **Relativity**

A [15 points] A particle of mass $\mathbf{m_1}$ decays at rest into two particles with rest masses $\mathbf{m_2}$ and $\mathbf{m_3}$, respectively. The $i^{th}$ particle has 4-momentum $\boldsymbol{p}_i = m_i \gamma_i \{1, 0, 0, \beta_i\}$, where and $\gamma_i=(1 - \beta_i^2)^{-1/2}$, i=1,2,3, and $\beta_1=0$.

i. [5 points] Give an expression for the relation between the three 4-vectors. What conservation law(s) does it represent?

ii. [5 points] Find the invariant mass of the two particle system following the decay.

iii. [5 points] Find the kinetic energy of particle 3 in terms of $m_1$, $m_2$, $m_3$ and $\beta_2$.

B [15 points] In system **S**, two events at coordinates $\mathbf{(x_1,y_1,z_1)}$ and $\mathbf{(x_2,y_2,z_2)}$ occur simultaneously at time $\mathbf{t_0}$. System **S'** moves at a velocity $\boldsymbol{\beta c}$ along the x axis with respect to system **S**. Show that the two events are not simultaneous in **S'**, but instead have a time separation of $\boldsymbol{\Delta t = -\beta \gamma \Delta x/c}$, where $\boldsymbol{\gamma = 1/(1 - \beta^2)^{1/2}}$ and $\boldsymbol{\Delta x = x_2 - x_1}$.

C [20 points] The CERN SPS accelerator produces a beam of $^{208}$Pb nuclei with a laboratory total energy of 158 GeV/nucleon. This beam strikes a fixed target of $^{208}$Pb atoms. (***Note:*** the rest mass of a nucleon is 0.938 GeV. Ignore effects of binding energy.)

i. [6 points] Find the Lorentz factor $\gamma$ of the beam.

ii. [7 points] For Pb-Pb collisions, find the invariant mass per nucleon of the system and the center of mass collision energy per nucleon.

iii. [7 points] If a collider produced colliding beams of lead ions at the same energy, what would be the center of mass collision energy per nucleon?

## 2. [50 points total] **Astrophysics**

A [25 points] ***Dark Matter***: How does the orbital velocity **v** of a bright star that is in a circular orbit around a cluster of galaxies depend on the orbit radius **r** and the system mass M, under the following assumptions?

i. [10 points.] Under the assumption that the total attractive mass $\mathbf{M_V}$ is in the form of visible stars that are distributed symmetrically within a radius $\mathbf{R_V} < r$.

ii. [10 points.] Under the assumption that the mass of visible stars is negligible and that the orbit occurs within a uniform sphere of dark matter of mass $\mathbf{M_D}$ with a radius $\mathbf{R_D > r}$.

iii. [5 points.] It is observed that for a number of bright stars orbiting at various distances around a galactic cluster, $v(r) \sim r$ and that $v^2 r >> G M_V$. Discuss these results in the context of evidence for dark matter.

B [25 points] ***Superluminal Objects.*** Studies of the quasar 3C345 using long-baseline radio telescopes have shown that the object emitted a radio-bright "fireball" that, in observations made between 1969 and 1976 separated from the main body of the quasar and moved an apparent distance of 56 light years during the observations. Taken at face value, the hot object would be traveling at 8 times the speed of light. Assume that the quasar is a distance **D** from the Earth and that the hot object is moving away from the quasar in the general direction of the Earth at a relativistic velocity $\boldsymbol{\beta c}$, with its direction of travel making a small angle $\boldsymbol{\theta}$ with the line of sight between the quasar and the Earth.

i. [8 points.] Calculate the actual separation distance **R** between the quasar and fireball and the apparent separation distance $\boldsymbol{\Delta x}$ from the quasar, as seen by an observer on Earth, as the hot object moves away from the quasar in the time interval **t**. Here $\boldsymbol{\Delta x}$ is the component of **R** perpendicular to the line of sight, and **t**, **R** and $\boldsymbol{\Delta x}$ are in the reference frame of an observer on Earth.

ii. [8 points.] Calculate the transit time $\mathbf{T_1}$ to the Earth of radio waves emitted when the object is ejected from the quasar, the transit time $\mathbf{T_2}$ of radio waves emitted by the hot object a time **t** later, and the time difference $\boldsymbol{\Delta T}$ between the arrival at Earth of the waves from the two events.

iii. [9 points.] Assuming that $\theta$ is a small angle, use appropriate approximations to estimate the apparent velocity **v** of the object. Explain how it can exceed the velocity of light.

## 3. [50 points total] **Nuclear Physics**

A [20 points.] The atom $^{27}$Si (Z=14, N=13) has a mass of 26.986704 *u* and its "mirror nucleus" twin $^{27}$Al (Z=13, N=14) has a mass of 26.981539 *u*. Assume that nuclear radii obey the volume rule $\mathbf{R = r_0 A^{1/3}}$, where **A** is the mass number and $\mathbf{r_0}$ is a constant. Estimate the value of $\mathbf{r_0}$ from the data given above.
[*Note:* the classical electron radius is $r_c = e^2/m_e c^2 = e^2/0.511$ MeV $= 2.8 \times 10^{-13}$ cm, and 1 *u* = 931.502 MeV/$c^2$.]

B [30 points.]. The nuclear shell model describes the neutrons and protons of a nucleus as moving in independent nuclear potential wells, one for neutrons and the other for protons, with the spin **S** and angular momentum **L** coupled by an interaction $H_{SO} = -2\alpha\ \mathbf{S \bullet L}$, where $\alpha$ is a positive constant.

i. [7 points] What is the difference in the shapes of the shell-model potential wells used for the neutrons and for the protons? Why must they be treated as separate, rather than putting all nucleons into one well?

ii. [7 points] Discuss the ordering of the first three shells for neutrons, and the number of neutrons that will close each shell.

iii. [7 points] The nucleus $^{16}$O is a closed-shell nucleus, in the sense that its neutron and proton shells are both closed. Explain why and give its spin and parity.

iv. [9 points] Consider the three Z=N-1 nuclei that have a closed proton shell and a single "neutron hole" in the similar neutron shell. The shell closures are for the first, second, and third closed proton shells. What are these nuclei and what are their spins and parities?