Autumn 2002 Qualifying Examination - Electromagnetism

IMPORTANT
You must answer Question 1 and EITHER answer Question 2 OR Question 3.

1. [**50** points total] (**Time Dependent Fields**)

The scalar and vector potentials resulting from an arbitrary localized source are

$$V(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\int d\tau' \, \frac{\rho(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|} \quad , \quad \mathbf{A}(\mathbf{r},t) \; = \; \frac{\mu_0}{4\pi}\int d\tau' \, \frac{\mathbf{j}(\mathbf{r}',t_r)}{|\mathbf{r}-\mathbf{r}'|} \quad .$$

A [**12** points] Carefully define the quantities μ_0, ε_0, ρ, $\mathbf{j}$, τ', $\mathbf{r}$, $\mathbf{r}'$ and t_r.

Two infinite, parallel, straight wires (wire 1 and wire 2) separated by a distance $3d$, lie parallel to the $\mathbf{z}$-axis. One carries a current $I(t)$ in the $+\mathbf{z}$-direction, while the other carries current $-I(t)$ in the $+\mathbf{z}$-direction, where

$$I(t) = q_0 \; \delta(t) \quad ,$$

where $\delta(t)$ is a Dirac-Delta function. A point P lies in the plane of the wires and in the $z = 0$ plane. It is a distance d from wire 1, and a distance $2d$ from the wire 2, as shown in Fig. 1.

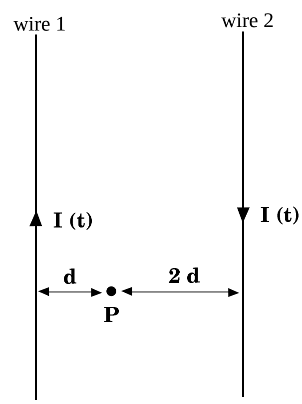

FIG. 1. *Two parallel wires carrying current $I(t)$ in opposite directions.*

B [**8** points] If an event occurs on wire 1 at a distance z along the $\mathbf{z}$-axis above the point P, how much time must pass before this event can influence physics at the point P? What is this time difference for an event on wire 2 with the same z-coordinate as the event on wire 1?

C [**18** points] What are the vector potential $\mathbf{A}(t)$ and scalar potential $V(t)$ at the point P due to $I(t)$, as a function of time?

D [**12** points] Determine the leading time-dependence of the electric field at the point P for $t \gg 2d/c$.

2. [**50** points total] (**Magnetostatics**)

A [**6** points] Write Maxwells equations for **D**, **E**, **H** and **B** in the presence of a free charge density ρ and a free current density **j**.

B [**5** points] In terms of the magnetic field, **B**, the magnetization **M** and the permeability of free space, define the field **H**.

C [**14** points] Consider an infinite straight wire in free-space carrying current I, located at $x = y = 0$ aligned in the z-direction. State Amperes law and use it to find the magnetic field **B** a distance r from the wire. Show that everywhere except at the wire, the field **H** can be written in terms of a scalar potential $\mathbf{H} = -\nabla\phi$, where (up to a constant)

$$\phi = -\frac{I}{2\pi}\,\mathrm{Im}\,[\,\log{(x+iy)}\,] \quad .$$

A long straight wire carrying current I is placed a distance d above a semi-infinite magnetic medium of permeability μ.

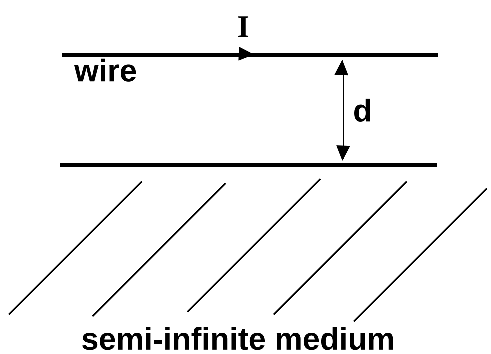

FIG. 2. *A long straight wire carrying current I a distance d above a semi-infinite magnetic medium of permeability μ.*

D [**10** points]

Given that there are no free charges or currents at the boundary between the magnetic medium and free-space, write down the relation between components of **B** just above the boundary and the components of **B** just below the boundary. Write this relation in terms of the scalar potential just above the boundary $\phi_>$, and just below the boundary $\phi_<$.

E [**15** points]

Find the force per unit length (including direction) on the wire using the method of images.

Formula that may be of use

$$\nabla = \hat{\mathbf{e}}_r\frac{\partial}{\partial r} \;+\; \hat{\mathbf{e}}_\theta\frac{1}{r}\frac{\partial}{\partial\theta} \;+\; \hat{\mathbf{e}}_z\frac{\partial}{\partial z} \quad .$$

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3. [**50** points total] (**Reflection, Refraction and Brewsters Angle**)

A monochromatic electromagnetic plane-wave polarized in the plane of incidence is incident upon a boundary between two linear, homogeneous media at an angle θ_I to the normal. Medium 1 has refractive index n_1 and medium 2 has refractive index n_2. Both media have permeabilities equal to that of the vacuum, $\mu_1 = \mu_2 = \mu_0$.

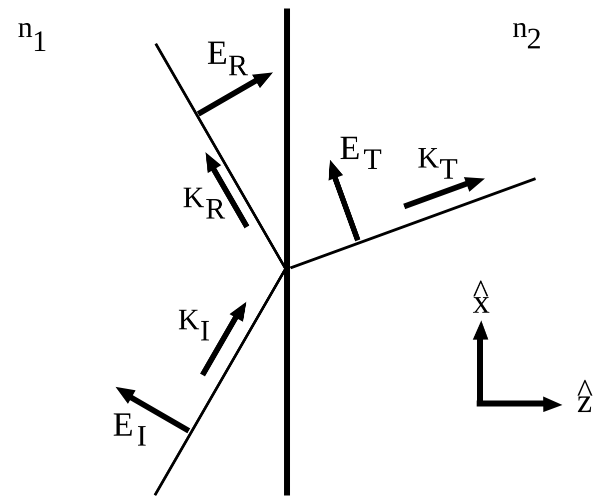

FIG. 3. *Dielectric media with refractive indices n_1 and n_2.*

A [**7** points] Write down the boundary conditions that relate arbitrary electric and magnetic fields on either side of the boundary.

B [**8** points] Write down general expressions for the incident, reflected and transmitted electric and magnetic fields. Indicate all spatial and time dependences.

C [**8** points] Derive relations between the reflected scattering angle θ_R and the incident angle θ_I, and between the transmitted angle θ_T and the incident angle θ_I .

D [**9** points] Relate the reflected electric field to the incident electric field.

E [**9** points] Find an expression for Brewsters angle, θ_B. What is θ_B for media with $n_1 = 1$ and $n_2 = 2.4$?

F [**9** points] Find the reflection and transmission coefficients.