

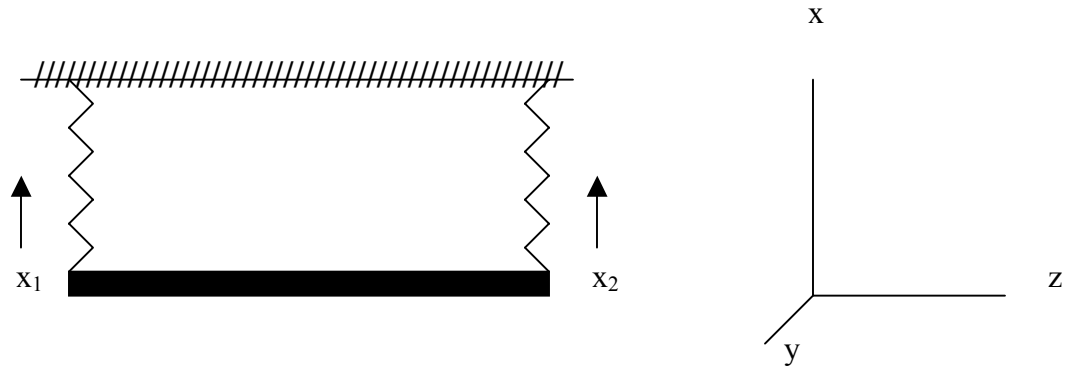
Autumn 2002 Qualifying Examination—Classical Mechanics

IMPORTANT: Please answer all three questions.

1. [30 points total] A point mass m moves in a circular orbit of radius r under the influence of a central force with **potential** $-km/r^n$.
 - A. [20 points] Find the conditions on n such that the circular orbit is stable under small perturbations (i.e., the mass will oscillate about the circular orbit).

 - B. [10 points] Find the frequency of small oscillations about stable circular orbits and express this frequency in terms of the angular velocity of the mass moving in these orbits.

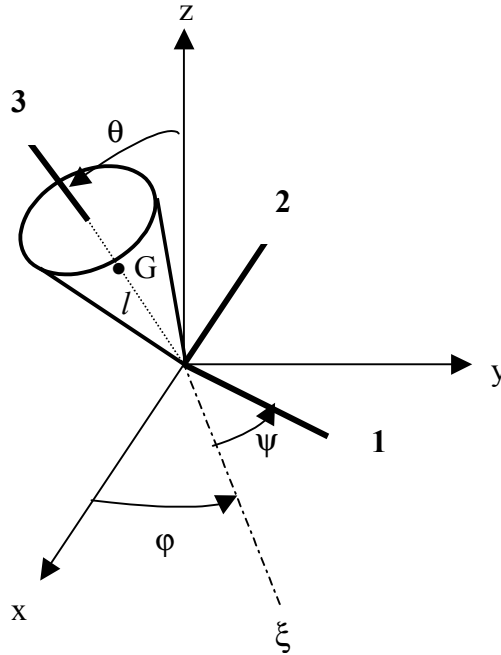
2. [40 points total] A rigid uniform bar of mass M and length L is supported in equilibrium in a horizontal position by two massless springs attached at each end.



The identical springs have the force constant k . The motion of the center of mass is constrained to move parallel to the vertical, x , axis. Furthermore the motion of the bar is constrained to lie in the xz -plane.

- A. [5 points] Show that the moment of inertia for a bar about the y axis through its center of mass is $ML^2/12$.
- B. [15 points] Construct the Lagrangian for this bar-spring arrangement assuming only small deviations from equilibrium.
- C. [15 points] Calculate the vibration frequencies of the normal modes for small amplitude oscillations.
- D. [5 points] Describe the normal modes of oscillation.

3. [30 points total] A symmetric top of mass M is in a uniform gravitational field along the z axis and has one fixed point at the tip.



The principal axes of the top are labeled 1, 2 and 3. The diagonal components of the inertia tensor in the principal axes are \mathbf{I}_1 , \mathbf{I}_2 and \mathbf{I}_3 , with $\mathbf{I}_1 = \mathbf{I}_2$ because of axial symmetry. The center of mass is located at the point labeled G, a distance of l from the tip.

A. [3 points] Show that the moment of inertia of the top about the **1**-axis in the above figure is

$$\mathbf{I} = \mathbf{I}_1 + Ml^2$$

B. [17 points] Show that the Lagrangian can be written in terms of the Euler angles, θ , ϕ and ψ as

$$L = \frac{1}{2} \mathbf{I} (\dot{\phi}^2 \sin^2 \theta + \dot{\theta}^2) + \frac{1}{2} \mathbf{I}_3 (\dot{\phi} \cos \theta + \dot{\psi})^2 - Mgl \cos \theta$$

C. [10 points] Show that components of the angular momentum about two axes are conserved.