Spring 2000 Qualifying Exam: Modern Physics

1. (20 points) The K^0 and its anti-particle $\overline{K}^0$ are neutral, spin-zero mesons of rest mass 498 MeV/c^2 that are coupled to both the strong and the weak interactions. The K^0 and $\overline{K}^0$ are typically produced in strong interactions, such as:

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

$$K^- + p \rightarrow \overline{K}^0 + n$$

The two mesons act differently in subsequent strong interactions, such as:

$$K^0 + p \rightarrow K^+ + n$$

$$\overline{K}^0 + p \rightarrow \pi^+ + \Lambda^0$$

a. (15 points) Given that the 4 interactions above take place, use a conservation law to explain why it is *not* possible for the strong interactions $K^0 + p \rightarrow \pi^+ + \Lambda^0$ and $\overline{K}^0 + p \rightarrow K^+ + n$ to take place.

b. (5 points) Explain why K^0 and $\overline{K}^0$ cannot decay by strong interactions.

2. (25 points) The neutral K does decay, relatively slowly, by the weak interactions. For weak decays it is useful to consider the CP eigenstates:

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$

The phases can be chosen such that $\mathrm{CP}|K^0\rangle = |\overline{K}^0\rangle$, and $\mathrm{CP}|\overline{K}^0\rangle = |K^0\rangle$.

a. (5 points) Find the CP eigenvalue for each of the above states.

b. (20 points) The most prominent decay channels of the neutral K mesons are the 2-pion modes, $\pi^+\pi^-$ and $\pi^0\pi^0$. Show that, to the extent CP is conserved, the state $|K_1\rangle$ can decay into two pions but $|K_2\rangle$ cannot. (Be sure to include the spatial wavefunction of the pions in your discussion, and to consider both 2-pion modes.)

3. (25 points) Because of its 2-pion decay mode, the state $|K_1\rangle$ has a much shorter lifetime than $|K_2\rangle$, which decays slowly into three pions. Therefore a beam of neutral K mesons in free space eventually becomes almost a pure $|K_2\rangle$ beam exhibiting very few 2-pion decays.

a. (15 points) If this nearly pure $|K_2\rangle$ beam then passes through a slab of material, the emerging beam is observed to produce many 2-pion decays. Explain carefully how this can happen.

b. (10 points) If instead the neutral K beam continues on in free space, the ratio of 2-pion decays to 3-pion decays does not approach zero, but is observed to level off at about $\epsilon = 2 \times 10^{-3}$. Explain the significance of this observation.

4. (30 points) Suppose that a K^0 is created in the lab with total energy 10 GeV. Answer the following questions, making reasonable approximations (of order 10% or better in the final answer) if you wish.

a. (10 points) If the K^0 lives for 1×10^{-10} s in its rest frame before decaying, how far does it travel in the lab?

b. (10 points) If the K^0 decays into two π^0 mesons having unequal energy in the lab, what is the smallest possible kinetic energy of the slower π^0? (The π^0 has a rest mass of 135 MeV/c^2.)

c. (10 points) If the K^0 decays into two π^0's of equal energy in the lab, what is the angle between the π^0 trajectories?