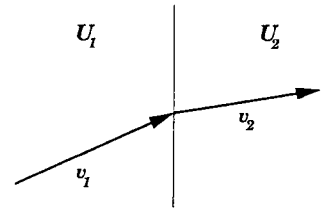


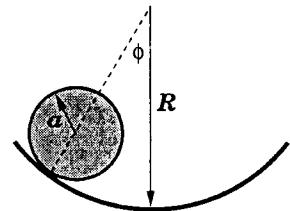
Spring '00 Qualifying Examination: Classical Mechanics

- A. (20 points) A particle of mass  $m$  and velocity  $\vec{v}_1$  leaves the region  $z < 0$  where its potential energy is a constant  $U_1$  and enters the region  $z > 0$  where its potential energy is a different constant  $U_2$ , as illustrated. Determine the change in direction of the particle.



- B. (40 points) A particle moves in three dimensions subject to the attractive central force  $\vec{F}(r) = -\left(\frac{\kappa}{r^2} + \frac{\eta}{r^4}\right)\hat{r}$ , with large angular momentum. (“Large” means greater than anything that’s relevant to compare it to in the following.)  $\kappa$  and  $\eta$  are positive constants;  $\hat{r}$  is a unit vector in the radial direction.
1. Find, and sketch, the effective radial potential. Show that it has a local maximum at a radius  $r_1$  and a local minimum at a radius  $r_2$ . What are  $r_1$  and  $r_2$ ?
  2. For what range of energy and initial position can the particle reach the origin but not reach infinity?
  3. For what range of energy and initial position will the motion of the particle remain bound, but never reach the origin?
  4. Describe, and sketch, the different possible types of trajectories for which the particle initially comes in from infinity.
  5. Describe, and sketch, the different possible types of trajectories for which the particle never reaches infinity as  $t \rightarrow \pm\infty$ .

- C. (40 points) A homogeneous cylinder of radius  $a$ , and total mass  $\mu$ , rolls without slipping (and without friction) inside a cylindrical surface with radius of curvature  $R$ , as shown.



1. What is the Lagrangian of this system?
2. If the cylinder is slightly displaced away from equilibrium at the bottom of the surface, what is the frequency of small oscillations?
3. As the amplitude of oscillation increases, does the frequency increase or decrease?