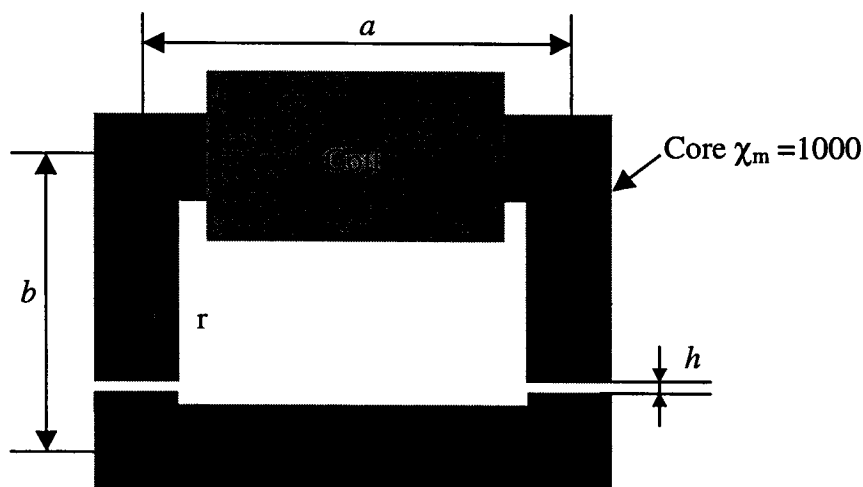


Qualifying Examination, Autumn '00

Electricity and Magnetism

1. [25 pts] An infinite charge sheet lies in the xy plane and carries a periodic surface charge density given by $\sigma = \sigma_0 \cos(kx)$. Calculate the electric potential produced by this charge distribution everywhere in space. Check your result for the case $k \rightarrow 0$.
2. [50 pts.] A U shape electromagnet designed for lifting metal objects is shown in the figure below. The dimensions of the magnet core are shown in the figure. The core has a circular cross-sectional area $A = \pi r^2$ and magnetic susceptibility $\chi_m = 1000$. Assume that the object being lifted has the same χ_m and A . The coil has N turns and a constant current I is flowing through it. The goal of this problem is to estimate the lifting force as a function of the gap h . Assume that $h \ll a, b, r$, but $\chi_m h$ is comparable to a, b .



- a) [10 pt] Write down the relationship between \mathbf{B} and \mathbf{H} in the core and in the gap.
 - b) [10 pt] Calculate the magnetic fields as a function of h (*hint*: use Ampere's law).
 - c) [10 pt] Calculate the energy of the electromagnetic field. Assume that the field is concentrated in the core and the gaps.
 - d) [10 pt] Calculate the work done by the power supply of the coil when the gap distance is changed. Assume that the power supply maintains a constant current.
 - e) [10 pt] Calculate the lifting force.
3. [25 pt] One might consider obtaining a limit on the electrical charge of the Earth based on the fact that its orbit around the Sun has been stable for about 4 billion years.
 - a) [10] Find an expression for the fractional loss rate of orbital kinetic energy due to electromagnetic radiation, assuming a circular orbit and a perfect vacuum.
 - b) [7] Use this to make an order of magnitude numerical estimate of the limit on the charge. (The mass of the Earth is 6×10^{24} kg).
 - c) [4] What would the electric field be near the surface of the Earth if the charge were at the limit you obtained? Is there evidence to suggest that the actual field is less than this?
 - d) [4] What physical effects for the real Earth and interplanetary medium that would tend to reduce the size of the charge?